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TITLE: BOWING COEFFICIENT REPRESENTATION OF  
CURVATURE OF GEOGRAPHIC FEATURES

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1 BOWING COEFFICIENT REPRESENTATION  
2 OF CURVATURE OF GEOGRAPHIC FEATURES

3 BACKGROUND OF THE INVENTION

4 The present invention relates to geographic data and more particularly, the present  
5 invention relates to a way to represent how much a linearly extending geographic feature  
6 (such as a road) curves by using a bowing coefficient.

7 Various new vehicle safety systems have been developed that provide warnings to  
8 the vehicle driver or that modify operation of the vehicle (or component thereof) based  
9 upon conditions around the vehicle or other factors. Examples of some of these new  
10 vehicle safety systems include automatic headlight aiming, automatic (or adaptive) cruise  
11 control, obstacle warning, curve warning, intersection warning, lane departure warning,  
12 collision warning, and adaptive transmission shift control. Some of these vehicle safety  
13 systems use sensor equipment (e.g., radar and cameras) to detect the current state of the  
14 roadway and environment around the vehicle. Some of these vehicle safety systems use  
15 digital map data as a component. Digital map data are used to identify the location and  
16 shape of the road ahead of and around the vehicle. Digital map data can also be used to  
17 identify relatively permanent objects or features along the roads.

18 Included among the types of digital map data used by some of these vehicle  
19 systems are data that indicate the curvature of the road. In some digital map databases,  
20 the curvature of a road at given location is indicated by a radius of curvature value (or  
21 inverse thereof). Some vehicle safety systems use these radius of curvature data to  
22 modify operation of the vehicle. For example, an automatic cruise control system in a  
23 vehicle uses the data that indicate curvature of a road at the location along the road at  
24 which a vehicle is traveling to determine an acceptable speed range for the vehicle. After  
25 determining an acceptable speed range, the automatic cruise control application adjusts  
26 the speed of the vehicle if necessary.

27 The map database used in this type of vehicle safety system includes data  
28 indicating the positions of points along roads including data indicating the radius of

1 curvature at the various points along the roads. When forming this type of map database,  
2 curvature values are computed using data that identify the coordinates of a series of  
3 points (referred to a "shape points") through which the road passes.

4 Although using computed values of radius of curvature to represent road shape is  
5 satisfactory for some vehicle applications, there is room for improvement. The computed  
6 radius of curvature values can be affected by the type of function (e.g., piecewise linear,  
7 b-spline, etc.) that is used to approximate the given set of shape points. Furthermore,  
8 small changes in the approximating function may produce large changes in the curvature  
9 value. Also, small changes in the data point locations themselves may produce a large  
10 change in a computed radius of curvature.

11 Accordingly, there is a need for another way to represent road geometry in a  
12 geographic database.

13

#### 14 SUMMARY OF THE INVENTION

15 To address these and other objectives, the present invention comprises a way to  
16 represent the curvature of a linearly extending feature, such as a road. The curvature of a  
17 linearly extending feature, such as a road, is represented using a bowing coefficient. The  
18 bowing coefficient at a given location along a linearly extending feature is determined by  
19 comparing the distance along the feature between two points on either side of the given  
20 location (or an approximation of the distance) to a straight-line distance between these  
21 same two points. Bowing coefficient data can be used by various vehicle systems that  
22 require information about the curvature of linearly extending features, such as roads upon  
23 which the vehicle is traveling.

24

#### 25 BRIEF DESCRIPTION OF THE DRAWINGS

26 Figure 1 is a diagram used to illustrate some of the terminology in this  
27 specification.

28 Figure 2 is a diagram used to illustrate the bowing coefficient at different  
29 locations along a road.

30 Figure 3 is a diagram that illustrates how the bowing coefficient varies with  
31 curvature.

1 Figure 5 is an enlarged portion of the area encompassed within the dotted circle in  
2 Figure 4.

3 Figure 6 is a diagram illustrating operation of an alternative embodiment.

4 Figure 7 is a diagram illustrating operation of another alternative embodiment.

5

## 6 DETAILED DESCRIPTION OF THE 7 PRESENTLY PREFERRED EMBODIMENTS

### 8 I. DEFINITIONS

9 The following terms are illustrated in Figure 1.

10 "Chord" refers to the straight-line segment joining any two points on a road  
11 segment.

12 "Chord length" refers to the length of the straight-line segment joining any two  
13 points on a road segment.

14 "Arc" refers to the portion of a road segment between any two points on the road  
15 segment.

16 "Arc length" refers to the length (or approximation thereof) of the portion of a  
17 road segment between any two points on the road segment.

18 As used in this specification, the word "curvature" refers to the general property  
19 of a linearly extending feature being curved and is not restricted to a mathematical  
20 definition, except as otherwise indicated. The word "curvature" in the phrase "radius of  
21 curvature" is understood to have its mathematical meaning.

22

### 23 II. OVERVIEW

24 Figure 2 shows a road segment 10. Several chords 12 have been drawn (in dashed  
25 lines) between pairs of points 14 on this segment 10. From Figure 2, it can be seen that  
26 the portions of the road segment, which are nearly straight, have arc lengths which are  
27 nearly equal to the corresponding chord lengths. Conversely, portions of the road  
28 segment that are curved have much larger arc lengths than corresponding chord lengths.  
29 This means that for relatively straight portions of a road segment, the ratio of arc length  
30 to chord length is close to 1, and for relatively curved portions of a road segment, the  
31 ratio of arc length to chord length is significantly greater than 1. In other words, the ratio  
32 of arc length to chord length is an indication of the local curvature of the road segment.

1           This relationship between curvature of a road segment, the chord length, and the  
2 arc length is described by the following.

$$\frac{A}{C} \sim \kappa$$

3  
4       where  $C$  is the chord length,  $A$  is the arc length and  $\kappa$  is the curvature (i.e., using the  
5 mathematical definition of "curvature").

6           For purposes of this specification, the ratio of  $A$  to  $C$  is referred to as the "bowing  
7 coefficient." The "bowing coefficient." is a measure of how much a portion of road  
8 segment between two points bends or bows out from the straight line joining these same  
9 two points.

10          Figure 3 illustrates the variation in the bowing coefficient with curvature.  
11 Figure 3 shows three alternative paths 30, 32, and 34 along a road segment 38. Each of  
12 these different paths represents a different alternative road configuration between the  
13 points 40 and 42. As shown by Figure 3, the greater the curvature, the greater the bowing  
14 coefficient.

### 15 16   III.   IMPLEMENTATION

17          The bowing coefficient can be used by various vehicle systems and applications  
18 that use data that represent road geometry. For example, the bowing coefficient can be  
19 used by an automatic cruise control application. Use of the bowing coefficient by an  
20 automatic cruise control application is described in connection with Figures 4 and 5.

21          In Figure 4, a vehicle 50 having an automatic cruise control system travels along a  
22 road segment 52. The automatic cruise control system obtains data indicating the  
23 position of the vehicle with respect to the road as represented by data contained in a map  
24 database. This function can be performed using known vehicle positioning technology,  
25 such as GPS, dead-reckoning, and so on.

26          As the vehicle 50 travels along the road segment 52, the automatic cruise control  
27 application in the vehicle 50 adjusts the speed of the vehicle based on the bowing  
28 coefficient of the road. In Figure 4, the arrows 54 indicate the instantaneous positions of  
29 the vehicle. The chords 58 corresponding to these positions are also shown.

1 According to one embodiment, at each position 54, the automatic cruise control  
2 application in the vehicle selects two points straddling the position at which the vehicle is  
3 located. The automatic cruise control application then computes the chord length  $C$ , the  
4 arc length  $A$  and the bowing coefficient using the two points. As indicated in Figure 5, as  
5 the vehicle moves into the curve, the bowing coefficient increases, and as the vehicle  
6 comes out of the curve the bowing coefficient decreases, as is expected. Using these  
7 computed values of the bowing coefficients, the automatic cruise control application in  
8 the vehicle adjusts the vehicle speed accordingly.

9 The selection of the two points that straddle the position of the vehicle is  
10 configurable so that the bowing coefficient derived therefrom is suitable for the  
11 application by which it is used. As an example, the distance (or distance range) of each  
12 of the two points from the position of the vehicle is configurable.

13 As illustrated in Figure 5, some of the arc lengths and chord lengths overlap for  
14 successive positions at which the bowing coefficient is determined. As previously  
15 indicated, the selection of the points used to determine the bowing coefficient is  
16 configurable so that an appropriate measure of the curvature is obtained. There is no  
17 constraint that the arc lengths and chord lengths not overlap.

18

#### 19 IV. ALTERNATIVE EMBODIMENTS

##### 20 A. Using previously calculated bowing coefficient data

21 According to one alternative embodiment, a vehicle safety system, such as  
22 automatic cruise control, uses a map database that includes bowing coefficients for points  
23 along roads. According to this alternative embodiment, the values for the bowing  
24 coefficient at points along roads are computed in advance by the database developer and  
25 stored in the geographic database. The geographic database, which includes the  
26 computed values for the bowing coefficient for points along roads, is installed in the  
27 vehicle and used by vehicle safety systems, such as automatic cruise control.

28 An advantage of storing computed values of the bowing coefficient in the map  
29 database used by the vehicle safety system is that it eliminates the need to compute these  
30 values in the vehicle.

31

1           B.     Calculating bowing coefficients

2           There are several different ways that the bowing coefficient can be calculated.  
3     One way to calculate a bowing coefficient is described in connection with Figure 6.

4           Figure 6 shows a portion of a road 60. Along the road are shape points 62, 64, 66,  
5     68, and 70. These shape points are points at which the geographic coordinates of the  
6     location of the road are known. The geographic coordinates at these locations may be  
7     determined using various data collection procedures. For example, the geographic  
8     coordinates at these positions may be determined using GPS equipment. Alternatively,  
9     the geographic coordinates at these positions may be determined from aerial photographs.

10          According to one alternative embodiment, a value of the bowing coefficient is  
11     determined for each shape point. According to this alternative embodiment, a value of  
12     the bowing coefficient at a given shape point can be approximated by comparing the sum  
13     of the distances from the given shape point to the two shape points immediately adjacent  
14     to the given shape point to the straight-line distance between the two adjacent shape  
15     points. According to this alternative, the arc length is approximated by using two "chord  
16     lengths."

17          For example, to determine the bowing coefficient for the shape point labeled 66,  
18     the distance from the shape point 64 to the shape point 66 is summed with distance from  
19     the shape point 66 to the shape point 68. Then, this sum is divided by the distance from  
20     the shape point 64 to the shape point labeled 68 in order to determine the bowing  
21     coefficient at the shape point 66. Bowing coefficients for the rest of the shape points can  
22     be determined in a similar manner. The values of the bowing coefficients can then be  
23     stored with the coordinates of the shape points in a geographic database. Alternatively,  
24     using this method, bowing coefficients can be computed on-the-fly, as needed, by an  
25     application in the vehicle during runtime.

26          According to another alternative, an approximation of the bowing coefficient at a  
27     given shape point can be determined by taking into account the distances to additional  
28     shape points beyond those immediately adjacent to the given shape point. Figure 7  
29     illustrates this embodiment. Figure 7 shows the same portion of road that is shown in  
30     Figure 6. In Figure 7 the bowing coefficient at the point 66 is approximated by first  
31     summing the straight-line distances from the point 62 to the point 64, the point 64 to the

1 point 66, the point 66 to the point 68, and the point 68 to the point 70. The sum of these  
2 distances is then divided by the straight-line distance between the point 62 and the point  
3 70 thereby yielding the bowing coefficient at the point 66. According to this alternative,  
4 the arc length is approximated by using four "chord lengths."

5 In Figure 7, two points were selected on either side of the point at which the  
6 bowing coefficient was determined. Alternatively, any number of points can be selected  
7 on either side of the point at which the bowing coefficient is determined.

8 According to still another alternative, the distances from the point along a road at  
9 which the bowing coefficient is determined to the points adjacent thereto used in  
10 determining the bowing coefficient can be actual distances as-the-vehicle-travels. The  
11 actual distances as-the-vehicle-travels can be collected using odometer or speed pulse  
12 data or determined from examination of aerial or satellite photographs.

13

#### 14 C. Other alternatives

15 In another alternative, the values of the bowing coefficient can be computed on  
16 the fly in the vehicle using data representing the road geometry, such as shape point data.

17 The new method described here can be used in combination with the existing  
18 radius of curvature approach to improve the robustness and effectiveness of various  
19 vehicle applications.

20 In the embodiments described above, bowing coefficient data were used to  
21 represent the curvature of roads. In alternative embodiments, bowing coefficient data can  
22 be used to represent the curvature of other kinds of linearly extending features, such as  
23 rivers, railroad tracks, boundaries, trajectories, ferries, and so on.

24

#### 25 V. ADVANTAGES

26 Several advantages follow from using bowing coefficients to represent curvature.  
27 First, using bowing coefficients to represent the curvature of linearly extending  
28 geographic features, such as roads, does not involve the computation of radius of  
29 curvature values which are prone to large errors. Further, using bowing coefficients to  
30 represent the curvature of linearly extending features is less computationally intensive  
31 than using radius of curvature values. In addition, bowing coefficients can be derived



1 from data that are stored as a set of shape points (piecewise linear approximation),  
2 polynomial spline control points, etc.

3

4 It is intended that the foregoing detailed description be regarded as illustrative  
5 rather than limiting and that it is understood that the following claims including all  
6 equivalents are intended to define the scope of the invention.